FUZZY GRADE OF SOME HYPERGROUPS

[1]P. CORSINI, [2]V. LEOREANU-FOTEA AND [1]M. A. LEPELLERE

[1]*Department of Civil Engineering and Architecture, University of Udine, Via delle Scienze 206, 33100 Udine, Italy*
[2]*Faculty of Mathematics, Al. I. Cuza University, Bd. Carol I, n. 11, Iasi, Romania*
piergiuliocorsini@gmail.com, maria.lepellere@uniud.it, foteavioleta@gmail.com

Abstract. One determines the fuzzy grade of a hypergroup of functions and of some hypergroups associated with ordered sets.

1. **Introduction**

Algebraic hypoerstructures have been developed not only from the theoretical point of view, but also for the applications they have in various fields, [2] - [25], [17]-[21]. The determination of the Fuzzy Grade of a hypergroup (or more generally of a hypergroupoid) is one of the most prominent problems of the theme: Connections between Fuzzy Sets and Hyperstructures. It has been considered for the first time by Corsini [3] in 1993 and it has been studied also in the following years by several scientists in Romania, Greece, Italy, Iran, Canada, [1], [3], [4] - [27], [22] - [26]. In this paper one has studied the Fuzzy Grade in two situations: for hypergroups of transformations of a set A, in the case $\mid A \mid = 2$ and for hypergroups associated with ordered sets of cardinality m such that $2 \leq m \leq 10$ and $m = 12$.

2. **Hypergroup of transformations of a set A, in the case $\mid A \mid = 2$.**

Set $A = \{1, 2\}$, and for $1 \leq i \leq 7$, let us define the following functions as follows:

$$f_1(1) = 1, f_1(2) = 2, f_2(1) = 2, f_2(2) = 1,$$
$$f_3(1) = 1, f_3(2) = A, f_4(1) = 2, f_4(2) = A,$$
$$f_5(1) = A, f_5(2) = 1, f_6(1) = A, f_6(2) = 2$$
$$f_7(1) = f_7(2) = A.$$

Let $\mathcal{F}$ be the set $\{f_i\}_{1 \leq i \leq 7}$. For all (i, j), set

$$f_i \circ f_j = \{h \in \mathcal{F} \mid \forall x \in A,\ h(x) \subseteq f_i(f_j(x))\}.$$

Received: ***; Revised:***; Accepted: ***
Key words and phrases: Ends lemma, EL–hyperstructures, quasi-ordered semigroups, partially ordered semigroups

We have the following structure:

H_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7
f_1	f_1	f_2	f_1, f_3	f_2, f_4	f_2, f_5	f_6, f_1	$\mathcal{F}$
f_2	f_2	f_1	f_2, f_4	f_1, f_3	f_1, f_6	f_2, f_5	$\mathcal{F}$
f_3	f_1, f_3	f_2, f_5	f_1, f_3	$\mathcal{F}$	f_2, f_5	$\mathcal{F}$	$\mathcal{F}$
f_4	f_2, f_4	f_1, f_6	f_2, f_4	$\mathcal{F}$	f_1, f_6	$\mathcal{F}$	$\mathcal{F}$
f_5	f_2, f_5	f_1, f_3	$\mathcal{F}$	f_1, f_3	$\mathcal{F}$	f_2, f_5	$\mathcal{F}$
f_6	f_1, f_6	f_2, f_4	$\mathcal{F}$	f_2, f_4	$\mathcal{F}$	f_1, f_6	$\mathcal{F}$
f_7	$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$

We have (see [3], [4])

$$A_1(f_1) = 2/1 + 12/2 + 21/7 = 11 = A_1(f_2); \quad q(f_1) = q(f_2) = 35.$$

So, $\mu_1(f_1) = \mu_1(f_2) = 0.314$. We have also

$$A_1(f_3) = A_1(f_4) = A_1(f_5) = A_1(f_6) = 6/2 + 21/7 = 6.$$

$$q(f_3) = q(f_4) = q(f_5) = q(f_6) = 27.$$

By consequence, $\forall i : 3 \leq i \leq 6, \quad \mu(f_i) = 0.222$ and we have also $A_1(f_7) = 21/7, \quad q(f_7) = 21$, so $\mu_1(f_7) = 1/7 = 0.143$.

So, we obtain the structure H_1.

Set $B = \{f_1, f_2\},\ C = \{f_3, f_4, f_5, f_6\}$. We find

H_1	f_1	f_2	f_3	f_4	f_5	f_6	f_7
f_1	B	B	$B \cup C$	$B \cup C$	$B \cup C$	$B \cup C$	A
f_2		B	$B \cup C$	$B \cup C$	$B \cup C$	$B \cup C$	A
f_3			C	C	C	C	$C \cup \{f_7\}$
f_4				C	C	C	$C \cup \{f_7\}$
f_5					C	C	$C \cup \{f_7\}$
f_6						C	$C \cup \{f_7\}$
f_7							f_7

We find

$$A_2(f_1) = A_2(f_2) = 4/2 + 16/6 + 4/7 = 5.241;$$

$$q_2(f_1) = q_2(f_2) = 24;\ \mu_2(f_1) = \mu_2(f_2) = 0.218$$

and

$$\forall i : 3 \leq i \leq 6,\ A_2(f_i) = 4 + 2.67 + 0.571 + 1.6 = 8.84,\ q_2(f_i) = 44,$$

so $\mu_2(f_i) = 0.201$.

Finally, $A_2(f_7) = 8/5 + 4/7 + 1 = 3.171,\ q_2(f_7) = 13,\ \mu_2(f_7) = 0.244$.

We obtain the structure H_2 :

H_2	f_3	f_4	f_5	f_6	f_1	f_2	f_7
f_3	C	C	C	C	$B\cup C$	$B\cup C$	A
f_4		C	C	C	$B\cup C$	$B\cup C$	A
f_5			C	C	$B\cup C$	$B\cup C$	A
f_6				C	$B\cup C$	$B\cup C$	A
f_1					B	B	$B\cup\{f_7\}$
f_2						B	$B\cup\{f_7\}$
f_7							f_7

We have

$$\forall i : 3 \leq i \leq 6,\ A_3(f_i) = 16/4 + 16/6 + 8/7 = 13.143,$$
$$q_3(f_i) = 40,\ \mu_3(f_i) = 0.329.$$

$$A_3(f_1) = A_3(f_2) = 4/2 + 16/6 + 8/7 = 11.143,$$
$$q_3(f_1) = q_3(f_2) = 28,\quad \mu_3(f_1) = \mu_3(f_2) = 0.398.$$

Finally,

$$A_3(f_7) = 1 + 8/7 + 4/3 = 3.476,\quad q_3(f_7) = 13,\ \mu_3(f_7) = 0.267.$$

Since

$$\mu_3(f_1) = \mu_3(f_2) > \mu_3(f_3) = \mu_3(f_4) = \mu_3(f_5) = \mu_3(f_6) > \mu_3(f_7),$$

we obtain a structure H_3 which coincides with H_1.

Let us consider now the set

$$\mathcal{F} = \{f \mid f : A \to \mathcal{P}^*(A) : \bigcup_{x\in A} f(x) = A\}$$

endowed with the hyperoperation $< \Box >$, defined as follows:

$$f\Box g = \{h \in \mathcal{F} \mid \forall x \in A,\ h(x) \subseteq f(x) \cup g(x)\}.$$

So, we find the following hyperstructure K_0 :

K_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7
f_1	f_1	$\mathcal{F}$	f_1, f_3	$\mathcal{F}$	$\mathcal{F}$	f_1, f_6	$\mathcal{F}$
f_2		f_2	$\mathcal{F}$	f_2, f_4	f_2, f_5	$\mathcal{F}$	$\mathcal{F}$
f_3			f_3, f_1	$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$
f_4				f_2, f_4	$\mathcal{F}$	$\mathcal{F}$	$\mathcal{F}$
f_5					f_2, f_5	$\mathcal{F}$	$\mathcal{F}$
f_6						f_1, f_6	$\mathcal{F}$
f_7							$\mathcal{F}$

We find

$$A_1(f_1) = A_1(f_2) = 1 + 6/2 + 35/7 = 9,\ q_1(f_1) = q_1(f_2) = 42,$$
$$\mu_1(f_1) = \mu_1(f_2) = 0.21428.$$

$$A_1(f_3) = A_1(f_4) = A_1(f_5) = A_1(f_6) = 1 + 2/2 + 35/7 = 7,$$
$$q_1(f_3) = q_1(f_4) = q_1(f_5) = q_1(f_6) = 38,$$
$$\mu_1(f_3) = \mu_1(f_4) = \mu_1(f_5) = \mu_1(f_6) = 0.1842.$$

Finally,

$$A_1(f_7) = 35/7 = 5,\ q_1(f_7) = 35,\ \mu_1(f_7) = 0.143.$$

We obtain a hyperstructure K_1 which coincides with H_1.

So, the grades of K_0 and H_0 coincide, that is $\partial K_0 = \partial H_0$.

3. **Hypergroups associated with ordered sets**

Given a totally ordered set $(H; <)$, the associated hypergroup $(H, \circ)$ is defined by (see [3], [4], [5]):

$$\forall (x, y) \in H^2,\ x \circ y = \{z \mid min\{x, y\} \leq z \leq max\{x, y\}\}.$$

The grade of H_2^0.

H_2^0	1	2
1	1	1, 2
2		2

$A_1(1) = A_1(2) = 1 + 2/2 = 2,$

$q_1(1) = q_1(2) = 3,$

$\mu_1(1) = \mu_1(2) = 0.666.$

So, H_2^1 is total and by consequence $\partial H_2^0 = 1$.

The grade of H_3^0.

H_3^0	1	2	3
1	1	1, 2	H
2		2	2, 3
3			3

$A_1(1) = A_1(3) = 1 + 2/2 + 2/3 = 8/3,$

$q_1(1) = q_1(3) = 5,$

$\mu_1(1) = \mu_1(3) = 0.5333$

$A_1(2) = 1 + 4/2 + 2/3 = 11/3;\ q_1(2) = 7;\ \mu_1(2) = 0.5238.$ Hence,

H_3^1	1	3	2
1	1, 3	1, 3	H
3		1, 3	H
2			2

$A_2(1) = A_2(3) = 4/2 + 4/3 = 10/3$

$q_2(1) = q_2(3) = 8,\ \mu_2(1) = \mu_2(3) = 0.4166$

$A_2(2) = 1 + 4/3 = 7/3;\ q_2(2) = 5;\ \mu_2(2) = 0.4666.$

We have clearly $H_3^2 = H_3^1$, by consequence $\partial H_3^0 = 1$.

The grade of H_4^0.

H_4^0	1	2	3	4
1	1	1, 2	1, 2, 3	H
2		2	2, 3	2, 3, 4
3			3	3, 4
4				4

$A_1(1) = A_1(4) = 1 + 2/2 + 2/3 + 2/4 = 19/6$

$q_1(1) = q_1(4) = 7$

$A_1(2) = A_1(3) = 1 + 4/2 + 4/3 + 2/4 = 29/6$

$q_1(2) = q_1(3) = 11$

$\mu_2(1) = \mu_1(4) = 0.4524,\ \mu_1(2) = \mu_1(3) = 0.4394.$

H_4^1	1	4	2	3
1	$1,4$	$1,4$	H	H
4		$1,4$	H	H
2			$2,3$	$2,3$
3				$2,3$

$A_2(1) = A_2(4) = A_2(2) = A_2(3) = 4/2 + 8/4 = 4$
$q_2(1) = q_2(4) = q_2(2) = q_2(3) = 12$
We find $\forall j,\ \mu_2(j) = 4/12 = 0.333$. By consequence, $\forall m \geq 2,\ H_4^m$ is total, so $\partial H_4^0 = 2$.

The grade of H_5^0.

H_5^0	1	2	3	4	5
1	1	$1,2$	$1,2,3$	$1,2,3,4$	H
2		2	$2,3$	$2,3,4$	$2,3,4,5$
3			3	$3,4$	$3,4,5$
4				4	$4,5$
5					5

$A_1(1) = 1 + 2/2 + 2/3 + 2/4 + 2/5 = 107/30,\ q_1(1) = 9,$
$\mu_1(1) = 0.3963 = \mu_1(5)$.

$A_1(2) = A_1(4) = 1 + 4/2 + 4/3 + 4/4 + 2/5 = 86/15,$
$q_1(2) = q_1(4) = 15,\ \mu_1(2) = \mu_1(4) = 0.3822;$

$A_1(3) = 1 + 4/2 + 6/3 + 4/4 + 2/5 = 96/15,$
$q_1(3) = 17,\ \mu_1(3) = 0.37647$. We have:

H_5^1	1	5	2	4	3
1	$1,5$	$1,5$	$1,5,2,4$	$1,5,2,4$	H
5		$1,5$	$1,5,2,4$	$1,5,2,4$	H
2			$2,4$	$2,4$	$2,3,4$
4				$2,4$	$2,3,4$
3					3

$A_2(1) = A_2(5) = 4/2 + 8/4 + 4/5 = 24/5,\ q_2(1) = q_2(5) = 16,$
$\mu_2(1) = 0.300 = \mu_2(5)$.

$A_2(2) = A_2(4) = 4/2 + 8/4 + 4/5 + 4/3 = 92/15,$
$q_2(2) = q_2(4) = 20,\ \mu_2(2) = \mu_2(4) = 0.3066;$

$A_2(3) = 1 + 4/3 + 4/5 = 47/15,$
$q_2(3) = 9,\ \mu_2(3) = 0.34815$. Hence

H_5^2	3	2	4	1	5
3	3	$2,3,4$	$2,3,4$	H	H
2		$2,4$	$2,4$	$2,4.1,5$	$2,4.1,5$
4			$2,4$	$2,4.1,5$	$2,4.1,5$
1				$1,5$	$1,5$
5					$1,5$

Therefore, $H_5^2 = H_5^1$, so $\partial H_5^0 = 1$.

The grade of H_6^0.

H_6^0	1	2	3	4	5	6
1	1	1, 2	1, 2, 3	1, 2, 3, 4	1, 2, 3, 4, 5	H
2		2	2, 3	2, 3, 4	2, 3, 4, 5	2, 3, 4, 5, 6
3			3	3, 4	3, 4, 5	3, 4, 5, 6
4				4	4, 5	4, 5, 6
5					5	5, 6
6						6

$A_1(1) = A_1(6) = 1 + 2/2 + 2/3 + 2/4 + 2/5 + 2/6 = 117/30$,
$q_1(1) = 11,\ \mu_1(1) = 0.3545 = \mu_1(6)$.

$A_1(2) = A_1(5) = 1 + 4/2 + 4/3 + 4/4 + 4/5 + 2/6 = 97/15$,
$q_1(2) = q_1(5) = 19,\ \mu_1(2) = \mu_1(5) = 0.34035$;

$A_1(3) = A_1(4) = 1 + 4/2 + 6/3 + 6/4 + 4/5 + 2/6 = 279/30$,
$q_1(3) = q_1(4) = 23,\ \mu_1(3) = \mu_1(4) = 0.3319$.

Hence

H_6^1	1	6	2	5	3	4
1	1, 6	1, 6	1, 6, 2, 5	1, 6, 2, 5	H	H
6		1, 6	1, 6, 2, 5	1, 6, 2, 5	H	H
2			2, 5	2, 5	2, 5, 3, 4	2, 5, 3, 4
5				2, 5	2, 5, 3, 4	2, 5, 3, 4
3					3, 4	3, 4
4						3, 4

$A_2(1) = A_2(6) = 4/2 + 8/4 + 8/6 = 32/6,\ q_2(1) = q_2(6) = 20$
$\mu_2(1) = \mu_2(6) = 0.2666$.

$A_2(2) = A_2(5) = 4/2 + 16/4 + 8/6 = 44/6$,
$q_2(2) = q_2(5) = 28,\ \mu_2(2) = \mu_2(5) = 0.2619$;

$A_2(3) = A_2(4) = 8/4 + 4/2 + 8/6 = 32/6$,
$q_2(3) = q_2(4) = 20,\ \mu_2(3) = \mu_2(4) = \mu_2(1) = \mu_2(6)$.

We obtain:

H_6^2	1	6	3	4	2	5
1	1, 6, 3, 4	1, 6, 3, 4	1, 6, 3, 4	1, 6, 3, 4	H	H
6		1, 6, 3, 4	1, 6, 3, 4	1, 6, 3, 4	H	H
3			1, 6, 3, 4	1, 6, 3, 4	H	H
4				1, 6, 3, 4	H	H
2					2, 5	2, 5
5						2, 5

For all $i \in \{1, 6, 3, 4\}$, we have
$A_3(i) = 16/4 + 16/6 = 40/6,\ q_3(i) = 32,\ \mu_3(i) = 0.2083$.

$A_3(2) = A_3(5) = 4/2 + 16/6 = 28/6$,
$q_3(2) = q_3(5) = 20,\ \mu_3(2) = \mu_3(5) = 0.2333$.

Then

H_6^3	1	6	3	4	2	5
1	1,6,3,4	1,6,3,4	1,6,3,4	1,6,3,4	H	H
6		1,6,3,4	1,6,3,4	1,6,3,4	H	H
3			1,6,3,4	1,6,3,4	H	H
4				1,6,3,4	H	H
2					2,5	2,5
5						2,5

We have clearly, $H_6^3 = H_6^2$, so $\partial H_6^0 = 2$.

The grade of H_7^0.

H_7^0	1	2	3	4	5	6	7
1	1	1,2	1,2,3	1,2,3,4	1,2,3,4,5	$H-\{7\}$	H
2		2	2,3	2,3,4	2,3,4,5	2,3,4,5,6	$H-\{1\}$
3			3	3,4	3,4,5	3,4,5,6	3,4,5,6,7
4				4	4,5	4,5,6	4,5,6,7
5					5	5,6	5,6,7
6						6	6,7
7							7

$A_1(1) = A_1(7) = 1 + 2/2 + 2/3 + 2/4 + 2/5 + 2/6 + 2/7 = 879/210$,
$q_1(1) = q_1(7) = 13,\ \mu_1(1) = 0.322 = \mu_1(7)$.

$A_1(2) = A_1(6) = 1 + 4/2 + 4/3 + 4/4 + 4/5 + 4/6 + 2/7 = 744/105$,
$q_1(2) = q_1(6) = 23,\ \mu_1(2) = \mu_1(6) = 0.308$;

$A_1(3) = A_1(5) = 6/3 + 1 + 4/2 + 6/4 + 6/5 + 4/6 + 2/7 = 1817/210$,
$q_1(3) = q_1(5) = 29,\ \mu_1(3) = \mu_1(5) = 0.2983$,
$A_1(4) = 1 + 4/2 + 6/3 + 8/4 + 6/5 + 4/6 + 2/7 = 1922/210$,
$q_1(4) = 31,\ \mu_1(4) = 0.2952$.

Hence

H_7^1	1	7	2	6	3	5	4
1	1,7	1,7	1,7,2,6	1,7,2,6	$H-\{4\}$	$H-\{4\}$	H
7		1,7	1,7,2,6	1,7,2,6	$H-\{4\}$	$H-\{4\}$	H
2			2,6	2,6	2,6,3,5	2,6,3,5	$H-\{1,7\}$
6				2,6	2,6,3,5	2,6,3,5	$H-\{1,7\}$
3					3,5	3,5	3,4,5
5						3,5	3,4,5
4							4

$A_2(1) = A_2(7) = 4/2 + 8/4 + 8/6 + 4/7 = 248/42$,

$q_2(1) = q_2(7) = 24,\ \mu_2(1) = 0.246 = \mu_2(7).$

$A_2(2) = A_2(6) = 4/2 + 16/4 + 8/6 + 4/5 + 4/7 = 1828/210,$
$q_2(2) = q_2(6) = 36,\ \mu_2(2) = \mu_2(6) = 0.2418;$

$A_2(3) = A_2(5) = 4/2 + 8/6 + 8/4 + 4/5 + 4/3 + 4/7 = 1688/210,$
$q_2(3) = q_2(5) = 32,\ \mu_2(3) = \mu_2(5) = 0.2512$

$A_2(4) = 1 + 4/3 + 4/5 + 4/7 = 389/105,$
$q_2(4) = 13,\ \mu_2(4) = 0.285.$

Therefore,

H_7^2	4	3	5	1	7	2	6
4	4	$3,4,5$	$3,4,5$	$H-\{2,6\}$	$H-\{2,6\}$	H	H
3		$3,5$	$3,5$	$3,5,1,7$	$3,5,1,7$	$H-\{4\}$	$H-\{4\}$
5			$3,5$	$3,5,1,7$	$3,5,1,7$	$H-\{4\}$	$H-\{4\}$
1				$1,7$	$1,7$	$1,7,2,6$	$1,7,2,6$
7					$1,7$	$1,7,2,6$	$1,7,2,6$
2						$2,6$	$2,6$
6							$2,6$

It is clear thet H_7^2 is isomorphic to H_7^1, so we can conclude that $\partial H_7^0 = 1$.

The grade of H_8^0.

H_8^0	1	2	3	4	5	6	7	8
1	1	$1,2$	$1,2,3$	$1,2$ $3,4$	$1,2,3$ $4,5$	$1,2,3$ $4,5,6$	$1,2,3,4$ $5,6,7$	H
2		2	$2,3$	$2,3,4$	$2,3$ $4,5$	$2,3,4$ $5,6$	$2,3,4$ $5,6,7$	$2,3,4,5$ $6,7,8$
3			3	$3,4$	$3,4,5$	$3,4$ $5,6$	$3,4,5$ $6,7$	$3,4,5$ $6,7,8$
4				4	$4,5$	$4,5$ 6	$4,5$ $6,7$	$4,5,6$ $7,8$
5					5	$5,6$	$5,6,7$	$5,6$ $7,8$
6						6	$6,7$	$6,7,8$
7							7	$7,8$
8								8

$A_1(1) = A_1(8) = 1 + 2/2 + 2/3 + 2/4 + 2/5 + 2/6 + 2/7 + 2/8 =$
$= 1863/420,\ q_1(1) = q_1(8) = 15,\ \mu_1(1) = 0.2957 = \mu_1(8).$

$A_1(2) = A_1(7) = 1 + 4/2 + 4/3 + 4/4 + 4/5 + 4/6 + 4/7+$
$2/8 = 3201/420,\ q_1(2) = q_1(7) = 27,\ \mu_1(2) = \mu_1(7) = 0.282275;$

$A_1(3) = A_1(6) = 1 + 4/2 + 6/3 + 6/4 + 6/5 + 6/6 + 4/7+$
$2/8 = 3999/420,\ q_1(3) = q_1(6) = 35,\ \mu_1(3) = \mu_1(6) = 0.2720.$

$A_1(4) = 1 + 4/2 + 6/3 + 8/4 + 8/5 + 6/6 + 4/7 + 2/8 =$
$= 4377/420 = A_1(5),$
$q_1(4) = q_1(5) = 39,\ \mu_1(4) = \mu_1(5) = 0.2672.$

H_8^1	1	8	2	7	3	6	4	5
1	1, 8	1, 8	1, 8 2, 7	1, 8 2, 7	1, 8, 2 7, 3, 6	1, 8, 2 7, 3, 6	H	H
8		1, 8	1, 8 2, 7	1, 8 2, 7	1, 8, 2 7, 3, 6	1, 8, 2 7, 3, 6	H	H
2			2, 7	2, 7	2, 7 3, 6	2, 7 3, 6	2, 7 3, 6 4, 5	2, 7 3, 6 , 4, 5
7				2, 7	2, 7 3, 6	2, 7 3, 6	2, 7 3, 6 4, 5	2, 7 3, 6 4, 5
3					3, 6	3, 6	3, 6 4, 5	3, 6 4, 5
6						3, 6	3, 6 4, 5	3, 6 4, 5
4							4, 5	4, 5
5								4, 5

$A_2(1) = A_2(8) = 4/2 + 8/4 + 8/6 + 8/8 = 38/6,$
$q_2(1) = q_2(8) = 28,\ \mu_2(1) = \mu_2(8) = \mu_2(4) = \mu_2(5) = 0.22619.$

$A_2(2) = A_2(7) = 4/2 + 16/4 + 16/6 + 8/8 = 58/6,$
$q_2(2) = q_2(7) = 44,\ \mu_2(2) = \mu_2(7) = \mu_2(3) = \mu_2(6) = 0.2197.$

H_8^2	1	8	4	5	2	7	3	6
1	$1,8$ $4,5$	$1,8$ $4,5$	$1,8$ $4,5$	$1,8$ $4,5$	H	H	H	H
8		$1,8$ $4,5$	$1,8$ $4,5$	$1,8$ $4,5$	H	H	H	H
4			$1,8$ $4,5$	$1,8$ $4,5$	H	H	H	H
5				$1,8$ $4,5$	H	H	H	H
2					$2,7$ $3,6$	$2,7$ $3,6$	$2,7$ $3,6$	$2,7$ $3,6$
7						$2,7$ $3,6$	$2,7$ $3,6$	$2,7$ $3,6$
3							$2,7$ $3,6$	$2,7$ $3,6$
6								$2,7$ $3,6$

We find $\forall j \in \{1, 2, ..., 9\}$,
$A_3(j) = 16/4 + 16/8 = 6$
$q_3(j) = 32$
$\mu_3(j) = 0.1875.$
So, H_8^3 is the total hypergroup.
Then $\partial H_8^0 = 3.$

The grade of H_9^0.

H_9^0	1	2	3	4	5	6	7	8	9
1	1	$1,2$	$1,2,3$	$1,2$ $3,4$	$1,2,3$ $4,5$	$1,2,3$ $4,5,6$	$1,2,3,4$ $5,6,7$	$1,2,3,4$ $5,6,7,8$	H
2		2	$2,3$	$2,3,4$	$2,3$ $4,5$	$2,3,4$ $5,6$	$2,3,4$ $5,6,7$	$2,3,4,5$ $6,7,8$	$2,3,4,5$ $6,7,8,9$
3			3	$3,4$	$3,4,5$	$3,4$ $5,6$	$3,4,5$ $6,7$	$3,4,5$ $6,7,8$	$3,4,5,6$ $7,8,9$
4				4	$4,5$	$4,5,6$	$4,5$ $6,7$	$4,5,6$ $7,8$	$4,5,6$ $7,8,9$
5					5	$5,6$	$5,6,7$	$5,6$ $7,8$	$5,6,7$ $8,9$
6						6	$6,7$	$6,7,8$	$6,7,8,9$
7							7	$7,8$	$7,8,9$
8								8	$8,9$
9									9

$A_1(1) = 1 + 2/2 + 2/3 + 2/4 + 2/5 + 2/6 + 2/7 + 2/8 + 2/9 =$
$= 5869/1260,\ q_1(1) = q_1(9) = 17,\ \mu_1(1) = 0.2740 = \mu_1(9).$

$A_1(2) = 1 + 4/2 + 4/3 + 4/4 + 4/5 + 4/6 + 4/7 + 4/8 + 2/9 =$
$= 10198/1260,\ q_1(2) = q_1(8) = 31,\ \mu_1(2) = \mu_1(8) = 0.2610;$

$A_1(3) = 1 + 4/2 + 6/3 + 6/4 + 6/5 + 6/6 + 6/7 + 4/8 + 2/9 =$
$12952/1260 = A_1(7),\ q_1(3) = q_1(7) = 41,\ \mu_1(3) = \mu_1(7) = 0.2507.$

$A_1(4) = 1 + 4/2 + 6/3 + 8/4 + 8/5 + 8/6 + 6/7 + 4/8 + 2/9 =$

$= 14506/1260 = A_1(6),\ q_1(4) = q_1(6) = 47,\ \mu_1(4) = \mu_1(6) = 0.24495.$

$A_1(5) = 1 + 4/2 + 6/3 + 8/4 + 10/5 + 8/6 + 6/7 + 4/8 + 2/9 =$
$= 15010/1260,\ q_1(5) = 49,\ \mu_1(5) = 0.243116.$

H_9^1	1	9	2	8	3	7	4	6	5
1	1, 9	1, 9	1, 9 2, 8	1, 9 2, 8	1, 9, 2 8, 3, 7	1, 9, 2 8, 3, 7	1, 9, 2, 8 3, 7, 4, 6	1, 9, 2, 8 3, 7, 4, 6	H
9		1, 9	1, 9 2, 8	1, 9 2, 8	1, 9, 2 8, 3, 7	1, 9, 2 8, 3, 7	1, 9, 2, 8 3, 7, 4, 6	1, 9, 2, 8 3, 7, 4, 6	H
2			2, 8	2, 8	2, 8 3, 7	2, 8 3, 7	2, 8, 3 7, 4, 6	2, 8, 3 7, 4, 6	2, 8, 3, 7 4, 6, 5
8				2, 8	2, 8 3, 7	2, 8 3, 7	2, 8, 3 7, 4, 6	2, 8, 3 7, 4, 6	2, 8, 3, 7 4, 6, 5
3					3, 7	3, 7	3, 7 4, 6	3, 7 4, 6	3, 7 4, 6, 5
7						3, 7	3, 7 4, 6	3, 7 4, 6	3, 7 4, 6, 5
4							4, 6	4, 6	4, 6, 5
6								4, 6	4, 6, 5
5									5

$A_2(1) = A_2(9) = 4/2 + 8/4 + 8/6 + 8/8 + 4/9 = 366/54,$
$q_2(1) = q_2(9) = 32,\ \mu_2(1) = 0.2118 = \mu_2(9).$

$A_2(2) = A_2(8) = 4/2 + 16/4 + 16/6 + 8/8 + 4/9 + 4/7 = 757/63,$
$q_2(2) = q_2(8) = 52,\ \mu_2(2) = \mu_2(8) = 0.23107;$

$A_2(3) = A_2(7) = 4/2 + 16/4 + 16/6 + 8/8 + 4/9 + 4/7 + 4/5 =$
$21702/1890,\ q_2(3) = q_2(7) = 56,\ \mu_2(3) = \mu_2(7) = 0.2050.$

$A_2(4) = 4/2 + 4/3 + 4/5 + 4/7 + 4/8 + 4/9 + 8/6 + 8/8 = 14142/1890$
$= A_2(6),\ q_2(4) = q_2(6) = 36,\ \mu_2(4) = \mu_2(6) = 0.207848.$

$A_2(5) = 1 + 4/3 + 4/5 + 4/7 + 4/9 = 1307/315,\ q_2(5) = 17,$
$\mu_2(5) = 0.24407.$

H_9^2	5	2	8	1	9	4	6	3	7
5	5	5 2, 8	5 2, 8	5, 2, 8 1, 9	5, 2, 8 1, 9	5, 2, 8 1, 9, 4, 6	5, 2, 8 1, 9, 4, 6	H	H
2		2, 8	2, 8	2, 8 1, 9	2, 8 1, 9	2, 8, 1 9, 4, 6	2, 8, 1 9, 4, 6	2, 8, 1, 9 4, 6, 3, 7	2, 8, 1, 9 4, 6, 3, 7
8			2, 8	2, 8 1, 9	2, 8 1, 9	2, 8, 1 9, 4, 6	2, 8, 1 9, 4, 6	2, 8, 1, 9 4, 6, 3, 7	2, 8, 1, 9 4, 6, 3, 7
1				1, 9	1, 9	1, 9 4, 6	1, 9 4, 6	1, 9, 4 6, 3, 7	1, 9, 4 6, 3, 7
9					1, 9	1, 9 4, 6	1, 9 4, 6	1, 9, 4 6, 3, 7	1, 9, 4 6, 3, 7
4						4, 6	4, 6	4, 6 3, 7	4, 6 3, 7
6							4, 6	4, 6 3, 7	4, 6 3, 7
3								3, 7	3, 7
7									3, 7

$A_3(1) = A_3(9) = 4/2 + 16/4 + 4/5 + 4/7 + 4/9 + 8/6 + 8/8 + 8/6$
$= 21702/1890,\ q_3(1) = q_3(9) = 56, \mu_3(1) = \mu_3(9) = 0.20504,$

$A_3(2) = A_3(8) = 4/2 + 4/3 + 8/4 + 4/5 + 4/7 + 4/9 + 8/6 + 8/8$
$= 17922/1890,\ q_3(2) = q_3(8) = 44,\ \mu_3(2) = \mu_3(8) = 0.2155,$

$A_3(3) = A_3(7) = 4/2 + 8/4 + 8/6 + 8/8 + 4/9 = 366/54,$
$q_3(3) = q_3(7) = 32, \mu_3(3) = \mu_3(7) = 0.21180,$

$A_3(4) = A_3(6) = 4/2 + 16/4 + 4/7 + 4/9 + 16/6 + 8/8$
$= 4038/378,\ q_3(4) = q_3(6) = 52,\ \mu_3(6) = \mu_3(4) = 0.205433,$

$A_3(5) = 1 + 4/3 + 4/5 + 4/7 + +4/9 = 1307/315,$
$q_3(5) = 17,\ \mu_3(5) = 0.24407.$

Let g be the function from H_1^9 to H_2^9 defined by

$$g(1) = 3, g(2) = 4, g(3) = 1, g(4) = 2,$$

$$g(5) = 5, g(6) = 8, g(7) = 9, g(8) = 6, g(9) = 7.$$

One can check that g is an isomorphism, so $\partial H_9^0 = 1$·

The grade of H_{10}^0.

H_{10}^0	1	2	3	4	5	6	7	8	9	10
1	1	1, 2	1, 2 3	1, 2 3, 4	1, 2 3, 4 5	1, 2 3, 4 5, 6	1, 2, 3 4, 5 6, 7	1, 2, 3 4, 5, 6 7, 8	1, 2, 3 4, 5, 6 7, 8, 9	H
2		2	2, 3	2, 3 4	2, 3 4, 5	2, 3, 4 5, 6	2, 3, 4 5, 6, 7	2, 3, 4, 5 6, 7, 8	2, 3, 4, 5 6, 7, 8, 9	$H - \{1\}$
3			3	3, 4	3, 4 5	3, 4 5, 6	3, 4, 5 6, 7	3, 4, 5 6, 7, 8	3, 4, 5, 6 7, 8, 9	3, 4, 5, 6 7, 8, 9, 10
4				4	4, 5	4, 5 6	4, 5 6, 7	4, 5, 6 7, 8	4, 5, 6 7, 8, 9	4, 5, 6 7, 8, 9, 10
5					5	5, 6	5, 6 7	5, 6 7, 8	5, 6, 7 8, 9	5, 6, 7 8, 9, 10
6						6	6, 7	6, 7 8	6, 7 8, 9	6, 7 8, 9, 10
7							7	7, 8	7, 8, 9	7, 8, 9, 10
8								8	8, 9	8, 9, 10
9									9	9, 10
10										10

$A_1(1) = 1 + 2/2 + 2/3 + 2/4 + 2/5 + 2/6 + 2/7 + 2/8 + 2/9 + 2/10 =$
$= 6121/1260 = A_1(10),\ q_1(1) = q_1(10) = 19,\ \mu_1(1) = 0.25568 = \mu_1(10).$

$A_1(2) = 1 + 4/2 + 4/3 + 4/4 + 4/5 + 4/6 + 4/7 + 4/8 + 4/9 + 2/10 =$
$= 10730/1260,\ q_1(2) = q_1(9) = 35,\ \mu_1(2) = \mu_1(9) = 0.2433;$

$A_1(3) = 1 + 4/2 + 6/3 + 6/4 + 6/5 + 6/6 + 6/7 + 6/8 + 4/9 + 2/10 =$
$13799/1260 = A_1(8),\ q_1(3) = q_1(8) = 47,\ \mu_1(3) = \mu_1(8) = 0.2330.$

$A_1(4) = 1 + 4/2 + 6/3 + 8/4 + 8/5 + 8/6 + 8/7 + 6/8 + 4/9 + 2/10 =$
$= 15713/1260 = A_1(7),\ q_1(4) = q_1(7) = 55,\ \mu_1(4) = \mu_1(7) = 0.2267.$

$A_1(5) = 1 + 4/2 + 6/3 + 8/4 + 10/5 + 10/6 + 8/7 + 6/8 + 4/9 + 2/10 =$
$= 16637/1260,\ q_1(5) = q_1(6) = 59,\ \mu_1(5) = \mu_1(6) = 0.2238.$

H_{10}^1	1	10	2	9	3	8	4	7	5	6
1	1, 10	1, 10	1, 10 2, 9	1, 10 2, 9	1, 10, 2 9, 3, 8	1, 10, 2 9, 3, 8	1, 10, 2, 4 9, 3, 8, 7	1, 10, 2, 4 9, 3, 8, 7	H	H
10		1, 10	1, 10 2, 9	1, 10 2, 9	1, 10, 2 9, 3, 8	1, 10, 2 9, 3, 8	1, 10, 2, 4 9, 3, 8, 7	1, 10, 2, 4 9, 3, 8, 7	H	H
2			2, 9	2, 9	2, 9 3, 8	2, 9 3, 8	2, 9 3, 8 4, 7	2, 9 3, 8 4, 7	2, 9, 3 8, 4, 7 5, 6	2, 9, 3 8, 4, 7 5, 6
9				2, 9	2, 9 3, 8	2, 9 3, 8	2, 9 3, 8 4, 7	2, 9 3, 8 4, 7	2, 9, 3 8, 4, 7 5, 6	2, 9, 3 8, 4, 7 5, 6
3					3, 8	3, 8	3, 8 4, 7	3, 8 4, 7	3, 8, 4 7, 5, 6	3, 8, 4 7, 5, 6
8						3, 8	3, 8 4, 7	3, 8 4, 7	3, 8, 4 7, 5, 6	3, 8, 4 7, 5, 6
4							4, 7	4, 7	4, 7 5, 6	4, 7 5, 6
7								4, 7	4, 7 5, 6	4, 7 5, 6
5									5, 6	5, 6
6										5, 6

We have
$A_2(1) = A_2(10) = 4/2 + 8/4 + 8/6 + 8/8 + 8/10 = 214/30$,
$q_2(1) = q_2(10) = 36$, $\mu_2(1) = 0.1981 = \mu_2(10) = \mu_2(5) = \mu_2(6)$.

$A_2(2) = A_2(9) = 4/2 + 16/4 + 16/6 + 16/8 + 8/10 = 344/30$,
$q_2(2) = q_2(9) = 60$, $\mu_2(2) = \mu_2(9) = 0.1911 = \mu_2(4) = \mu_2(7)$;

$A_2(3) = A_2(8) = 4/2 + 16/4 + 24/6 + 16/8 + 8/10 = 464/30$,
$q_2(3) = q_2(8) = 68$, $\mu_2(3) = \mu_2(8) = 0.2274$.

H_{10}^2	1	10	5	6	2	9	4	7	3	8
1	1, 10 5, 6	1, 10 5, 6	1, 10 5, 6	1, 10 5, 6 9, 4, 7	1, 10, 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	H	H
10		1, 10 5, 6	1, 10 5, 6	1, 10 5, 6 9, 4, 7	1, 10, 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	H	H
5			1, 10 5, 6	1, 10 5, 6	1, 10, 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	H	H
6				1, 10 5, 6	1, 10, 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	1, 10 5, 6, 2 9, 4, 7	H	H
2					2, 9 4, 7	2, 9 4, 7	2, 9 4, 7	2, 9 4, 7	2, 9, 4 7, 3, 8	2, 9, 4 7, 3, 8
9						2, 9 4, 7	2, 9 4, 7	2, 9 4, 7	2, 9, 4 7, 3, 8	2, 9, 4 7, 3, 8
4							2, 9 4, 7	2, 9 4, 7	2, 9, 4 7, 3, 8	2, 9, 4 7, 3, 8
7								2, 9 4, 7	2, 9, 4 7, 3, 8	2, 9, 4 7, 3, 8
3									3, 8	3, 8
8										3, 8

$A_3(1) = A_3(10) = A_3(5) = A_3(6) = 16/4 + 32/8 + 16/10 = 96/10$
$q_3(1) = q_3(10) = q_3(5) = q_3(6) = 64$,
$\mu_3(1) = \mu_3(10) = \mu_3(5) = \mu_3(6) = 0.15$.

$A_3(2) = A_3(9) = A_3(4) = A_3(7) = 16/4 + 32/8 + 16/4 + 16/6 + 16/10 = 976/60$
$q_3(2) = q_3(9) = q_3(4) = q_3(7) = 96$,
$\mu_3(2) = \mu_3(9) = \mu_3(4) = \mu_3(7) = 0.1694$.

$A_3(3) = A_3(8) = 4/2 + 16/6 + 16/10 = 376/60$, $q_3(3) = q_3(8) = 36$,
$\mu_3(3) = \mu_3(8) = 0.1741$.

So, we find a hyperstructure H_{10}^3 such that $H_{10}^3 = H_{10}^2$. By consequence, $\partial H_{10}^0 = 2$.

The grade of H_{12}^0.

H^0_{12}	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1 2	1 2 3	1, 2 3 4	1, 2 3, 4 5	1, 2 3, 4 5, 6	1, 2 3, 4 5, 6 7	1, 2 3, 4 5, 6 7, 8	1, 2, 3 4, 5 6, 7 8, 9	1, 2, 3 4, 5, 6 7, 8, 9 10	1, 2, 3, 4 5, 6, 7, 8 9, 10 11	H
2		2	2 3	2 3 4	2, 3 4, 5	2, 3 4, 5 6	2, 3 4, 5 6, 7	2, 3, 4 5, 6 7, 8	2, 3, 4 5, 6, 7 8, 9	2, 3, 4 5, 6, 7 8, 9, 10	2, 3, 4, 5 6, 7, 8 9, 10, 11	2, 3, 4, 5 6, 7, 8, 9 10, 11, 12
3			3	3, 4	3, 4 5	3, 4 5, 6	3, 4 5, 6 7	3, 4 5, 6 7, 8	3, 4, 5 6, 7 8, 9	3, 4, 5 6, 7, 8 9, 10	3, 4, 5 6, 7, 8, 9 10, 11	3, 4, 5, 6 7, 8, 9, 10 11, 12
4				4	4, 5	4, 5 6	4, 5 6 7	4, 5 6, 7 8	4, 5 6, 7 8, 9	4, 5, 6 7, 8, 9 10	4, 5, 6 7, 8, 9 10, 11	4, 5, 6, 7 8, 9, 10 11, 12
5					5	5, 6	5, 6 7	5, 6 7, 8	5, 6 7, 8 9	5, 6, 7 8, 9 10	5, 6, 7, 8 9, 10 11	5, 6, 7, 8 9, 10 11, 12
6						6	6, 7	6, 7 8	6, 7 8, 9	6, 7 8, 9 10	6, 7, 8 9, 10 11	6, 7, 8 9, 10 11, 12
7							7	7, 8	7, 8 9	7, 8 9, 10	7, 8, 9 10, 11	7, 8, 9, 10, 11, 12
8								8	8, 9	8, 9 10	8, 9 10, 11	8, 9, 10 11, 12
9 9									9	9, 10	9, 10 11	9, 10 11, 12
10										10	10 11	10, 11 12
11											11	11, 12
12												12

$A_1(1) = 1+2/2+2/3+2/4+2/5+2/6+2/7+2/8+2/9+2/10+2/11+2/12 = 144322/27720 = A_1(12)$,
$q_1(1) = q_1(12) = 23,\ \mu_1(1) = 0.226366 = \mu_1(12)$.

$A_1(2) = 1+4/2+4/3+4/4+4/5+4/6+4/7+4/8+4/9+4/10+4/11+2/12 = 256304/27720 = A_1(11)$,
$q_1(2) = q_1(11) = 43,\ \mu_1(2) = \mu_1(11) = 0.2150$;

$A_1(3) = 1+4/2+6/3+6/4+6/5+6/6+6/7+6/8+6/9+6/10+4/11+2/12 = 335526/27720 = A_1(10)$,
$q_1(3) = q_1(10) = 59,\ \mu_1(3) = \mu_1(10) = 0.205154$;

$A_1(4) = 1+4/2+6/3+8/4+8/5+8/6+8/7+8/8+8/9+6/10+4/11+2/12 = 390724/27720 = A_1(9)$,
$q_1(4) = q_1(9) = 71,\ \mu_1(4) = \mu_1(9) = 0.198526$;

$A_1(5) = A_2(8) = 1+4/2+6/3+8/4+10/5+10/6+10/7+10/8+8/9+6/10+4/11+2/12 = 425902/27720$
$q_1(5) = q_1(8) = 79,\ \mu_1(5) = \mu_1(8) = 0.194486$.

$A_1(6) = A_2(7) = 1+4/2+6/3+8/4+10/5+12/6+12/7+10/8+8/9+6/10+4/11+2/12 = 443062/27720$
$q_1(6) = q_1(7) = 83,\ \mu_1(6) = \mu_1(7) = 0.19257$.

H_{12}^1	1	12	2	11	3	10	4	9	5	8	6	7
1	1 12	1 12	1 12 2 11	1 12 2 11	1, 12 2, 11 3, 10	1, 12 2, 11 3, 10	1, 12 2, 11 3, 10 4, 9	1, 12 2, 11 3, 10 4, 9	1, 12 2, 11 3, 10, 4 9, 5, 8	1, 12 2, 11 3, 10, 4 9, 5, 8	H	H
12		1 12	1 12 2 11	1 12 2 11	1, 12 2, 11 3, 10	1, 12 2, 11 3, 10	1, 12 2, 11 3, 10 4, 9	1, 12 2, 11 3, 10 4, 9	1, 12 2, 11 3, 10, 4 9, 5, 8	1, 12 2, 11 3, 10, 4 9, 5, 8	H	H
2			2, 11	2, 11	2, 11 3, 10	2, 11 3, 10	2, 11 3, 10 4, 9	2, 11 3, 10 4, 9	2, 11 3, 10, 4 9, 5, 8	2, 11 3, 10, 4 9, 5, 8	2, 11 3, 10 4, 9, 5 6, 7, 8	2, 11 3, 10 4, 9, 5 6, 7, 8
11				2, 11	2, 11 3, 10	2, 11 3, 10	2, 11 3, 10 4, 9	2, 11 3, 10 4, 9	2, 11 3, 10, 4 9, 5, 8	2, 11 3, 10, 4 9, 5, 8	2, 11 3, 10 4, 9, 5 6, 7, 8	2, 11 3, 10 4, 9, 5 6, 7, 8
3					3, 10	3, 10	3, 10 4, 9	3, 10 4, 9	3, 10 4, 9 5, 8	3, 10 4, 9 5, 8	3, 10 4, 9, 5 8, 6, 7	3, 10 4, 9, 5 8, 6, 7
10						3, 10	3, 10 4, 9	3, 10 4, 9	3, 10 4, 9 5, 8	3, 10 4, 9 5, 8	3, 10 4, 9, 5 8, 6, 7	3, 10 4, 9, 5 8, 6, 7
4							4, 9	4, 9	4, 9 5, 8	4, 9 5, 8	4, 9, 5 8, 6, 7	4, 9, 5 8, 6, 7
9								4, 9	4, 9 5, 8	4, 9 5, 8	4, 9, 5 8, 6, 7	4, 9, 5 8, 6, 7
5									5, 8	5, 8	5, 8 6, 7	5, 8 6, 7
8										5, 8	5, 8 6, 7	5, 8 6, 7
6											6, 7	6, 7
7												6, 7

$A_2(1) = A_2(12) = A_2(6) = A_2(7) = 4/2{+}8/4{+}8/6{+}8/8{+}8/10{+}8/12 = 936/120$,
$q_2(1) = q_2(12) = q_2(6) = q_2(7) = 44$,
$\mu_2(1) = \mu_2(12) = \mu_2(6) = \mu_2(7) = 0.17727$.

$A_2(2) = A_2(11) = A_2(5) = A_2(8) = 4/2 + 16/4 + 16/6 + 16/8 + 16/10 + 8/12 = 1552/120$,
$q_2(2) = q_2(11) = q_2(5) = q_2(8) = 76$,
$\mu_2(2) = \mu_2(11) = \mu_2(5) = \mu_2(8) = 0.1702$;

$A_2(3) = A_2(10) = A_2(4) = A_2(9) = 4/2 + 16/4 + 24/6 + 24/8 + 16/10 + 8/12 = 1832/120$,
$q_2(3) = q_2(10) = q_2(4) = q_2(9) = 92$,
$\mu_2(3) = \mu_2(10) = \mu_2(4) = \mu_2(9) = 0.16594$.

Set $K_1 = \{1, 12, 6, 7\}$, $K_2 = \{2, 11, 5, 8\}$, $K_3 = \{3, 10, 4, 9\}$.

H_{12}^2	1	12	6	7	2	11	5	8	3	10	4	9
1	K_1	K_1	K_1	K_1	K_1 K_2	K_1 K_2	K_1 K_2	K_1 K_2	H	H	H	H
12		K_1	K_1	K_1	K_1 K_2	K_1 K_2	K_1 K_2	K_1 K_2	H	H	H	H
6			K_1	K_1	K_1 K_2	K_1 K_2	K_1 K_2	K_1 K_2	H	H	H	H
7				K_1	K_1 K_2	K_1 K_2	K_1 K_2	K_1 K_2	H	H	H	H
2					K_2	K_2	K_2	K_2	K_2 K_3	K_2 K_3	K_2 K_3	K_2 K_3
11						K_2	K_2	K_2	K_2 K_3	K_2 K_3	K_2 K_3	K_2 K_3
5							K_2	K_2	K_2 K_3	K_2 K_3	K_2 K_3	K_2 K_3
8								K_2	K_2 K_3	K_2 K_3	K_2 K_3	K_2 K_3
3									K_3	K_3	K_3	K_3
10										K_3	K_3	K_3
4											K_3	K_3
9												K_3

$A_3(1) = A_3(12) = A_3(6) = A_3(7) = A_3(3) = A_3(10) = A_3(4) = A_3(9) = 64/8 + 64/12 = 320/24$
$q_3(1) = q_3(12) = q_3(6) = q_3(7) = q_3(3) = q_3(10) = q_3(4) = q_3(9) = 128$,
$\mu_3(1) = \mu_3(12) = \mu_3(6) = \mu_3(7) = \mu_3(3) = \mu_2(10) = \mu_3(4) = \mu_3(9) = 0.104$.

$A_3(2) = A_3(11) = A_3(5) = A_3(8) = 16/4 + 64/12 = 112/12$
$q_3(2) = q_3(11) = q_3(5) = q_3(8) = 80$,
$\mu_3(2) = \mu_3(11) = \mu_3(5) = \mu_3(8) = 0.11667$.

H_{12}^3	1	12	6	7	3	10	4	9	2	11	5	8
1	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	H	H	H	H
12		K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	H	H	H	H
6			K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	H	H	H	H
7				K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	H	H	H	H
3					K_1 K_3	K_1 K_3	K_1 K_3	K_1 K_3	H	H	H	H
10						K_1 K_3	K_1 K_3	K_1 K_3	H	H	H	H
4							K_1 K_3	K_1 K_3	H	H	H	H
9								K_1 K_3	H	H	H	H
2									K_2	K_2	K_2	K_2
11										K_2	K_2	K_2
5											K_2	K_2
8												K_2

We have clearly $\mu_4 = \mu_3$, so $H_4 = H_3$. By consequence, $\partial H_{12}^0 = 3$.

4. **Conclusion**

From the above results, it seems it is possible to infer that

- if $n = 2$ or n is odd, then $\partial H_n^0 = 1$;
- if $n = 2p$, where p is a prime number, then $\partial H_n^0 = 2$;
- if $n = 2^2p$, where p is a prime number, then $\partial H_n^0 = 3$.

.

References

[1] R. Ameri, M.M. Zahedi, *Hypergroup and join space induced by a fuzzy subset,* PU.M.A., **8(2-3-4)** (1997)

[2] P. Corsini, *Prolegomena of Hypergroup Theory*, Aviani Editore, (1993)

[3] P. Corsini, *Join Spaces, Power Sets, Fuzzy Sets*, Proc. Fifth International Congress on A.H.A., 1993, Iasi, Romania

[4] P. Corsini, *A new connection between hypergroups and fuzzy sets*, Southeast Bulletin of Math., **27** (2003)

[5] P. Corsini, *Hyperstructures associated with ordered sets*, Bulletin of the Greek Mathematical Society, **48**, (2003)

[6] P. Corsini, *Join Spaces, multivalued functions and soft sets*, Proceedings of the International Conference on Algebra 2010, (ICA 2010), Universitas Gadjah Mada and the Southeast Asian Math.

[7] P. Corsini, I. Cristea, *Fuzzy grade of i.p.s. hypergroups of order less or equal to 6*, PU.M.A., **14(4)** (2003)

[8] P. Corsini, I. Cristea, *Fuzzy grade of i.p.s. hypergroups of order 7*, Iran J. of Fuzzy Systems, **1** (2004)

[9] P. Corsini, I. Cristea, *Fuzzy sets and non complete 1-hypergroups*, An. St. Univ. Ovidius Constanta, **13(1)** (2005)

[10] P. Corsini, B. Davvaz, *New connections among multivalued functions, hyperstructures and fuzzy sets*, Jordan Journal of Mathematics and Statistics, (JJMS), **3(3)** (2010)

[11] P. Corsini, V. Leoreanu, *Join Spaces associated with Fuzzy Sets*, J. of Combinatorics, Information and System Sciences, **20(1)**, (1995)

[12] P. Corsini, V. Leoreanu, *Applications of Hyperstructure Theory*, Advances in Mathematics, Kluwer Academic Publishers, (2003)

[13] P. Corsini, V. Leoreanu-Fotea, *On the grade of a sequence of fuzzy sets and join spaces determined by a hypergraph*, Southeast Asian Bulletin of Mathematics, **34** (2010).

[14] P. Corsini, V. Leoreanu-Fotea, A. Iranmanesh, *On the sequence of join spaces and membership functions determined by a hypergraph*, Journal of Multivalued-Logic and Soft Computing, **14(6)** (2008)

[15] P. Corsini, R. Mahjoob, *Multivalued functions, fuzzy subsets and join spaces*, Ratio Mathematica, **20** (2010)

[16] I. Cristea *A property of the connection between fuzzy sets and 32 hypergroupoids*, Italian Journal of Pure and Applied Mathematics, **21** (2007)

[17] S. Hoskova, P. Chvalina, C. Rackova, *Noncommutative join spaces of integral operators and related hyperstructures*, Advances MT, 1 (2000).

[18] J. Jantosciak, *Transposition hypergroups: noncommutative join spaces,* J.Algebra, **187** (1997).

[19] A. Maturo, I. Tofan, *Iperstrutture, strutture fuzzy ed applicazioni*, monografia di 168 pagine, pubblicazione finanziata con i fondi del progetto internazionale Socrates-Erasmus 2000/2001, Italia-Romania, Dierre Edizioni San Salvo, agosto

[20] W. Prenowitz, J. Jantosciak, *Join Geometries*, Springer-Verlag UTM, (1979)

[21] S.J. Rasovic, *Hyperrings constructed by multiendomorphisms of hypergroups*, Proceedings of the 10th International Congress on AHA, Brno, (2008)

[22] K. Serafimidis, A. Kehacias, M. Konstantinidou, *The L-fuzzy Corsini join hyperoperation*, Italian Journal of Pure and Applied Mathematics, **12** (2003)

[23] S. Spartalis, *The hyperoperation relation and the Corsini's partial or not partial hypergroupoid*, Italian Journal of Pure and Applied Mathematics, **24** (2008)

[24] M. Stefanescu, I. Cristea, *On the fuzzy grade of hypergroups*, Fuzzy Sets and Systems, **159** (2008)

[25] T. Vougiouklts, *Hyperstructures and their representations*, Hadronic Press Inc. (1994)

[26] M. Yavari, *Corsini's method and construction of join spaces*, Italian Journal of Pure and Applied Mathematics, **23** (2008)

[27] F. Yuming, *Algebraic hyperstructures obtained from algebraic structures with binary fuzzy binary relations*, Italian Journal of Pure and Applied Mathematics, **25** (2009)