ROOTS OF SCIENTIFIC THOUGHT IN THE ANCIENT GREEK WORLD

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ABSTRACT. The origins of mathematical thought in the Greek world date back, at least, to the V century B.C. However, only testimonials, but not written texts, survive of such origins. The first known mathematical texts are those of Euclid, Archimedes and Apollonius, written in the III century. Such works are written in a strictly technical language, the knowledge of which is necessary in order to understand them. For this reason, the main interpretative tradition of the ancient scientific thought was founded, for a long time, on the numerous commentaries flourished in the Roman imperial age, when the fertile phase of ancient science was exhausted. The Platonic or Aristotelian philosophy was then seen as foundation of all mathematical and scientific ideas. The first one was supported by the new-platonic schools, the second one used to explain the rigorous deductive structure of the scientific treatises. Neugebauer, at first, and successively Knorr, in the second half of twentieth century, put into question the influence of Plato and the philosophical thought in the work of the ancient Greek scientists. However, many questions remain open and there are still divergent and controversial positions. William Netz [36] , for example, describes Archimedes as an isolated genius who wrote only for future centuries. But a scientific discourse can't be made without an existing shared language and an existing shared conceptual system, that is, without an existing scientific paradigm. In the works of Euclid and Archimedes the author tracks down the signs and the characters of a very advanced and extraordinarily fruitful paradigm, but that, in the subsequent centuries, disappears and becomes unintelligible. What causes such disappearance? The author hypothesizes that from the second to the first century b.C. there was an interruption in the oral and direct transmission by teaching in schools. A *scientific paradigm*, in fact, can't be transmitted by philosophical written discourse, but only in a direct communication, in a training of research within a direct relationship between teacher and student.

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1. **Introduction**

That the modern Sciences sink their roots in the ancient Greek world is a common notion. But what this means, and which characters the ancient sciences really had, is far from clear. An extensive literature exists on this subject. A lot of testimonies refer to such origins that go from Thales and Pythagoras to the predecessors of Euclid as Eudoxus, but their informations are technically insufficient and always functionally inserted in a philosophical discourse.

The first scientific texts that survived until our time are the ones of Euclid. These are written in a strict technical language, without an adequate explanation of the meanings of the terms that are used. This is a fundamental point, because geometrical entities are abstract objects that are not corresponding to any object in the usual language and in the world of our real experience. So the interpretation of such terms became very difficult and was filtered, historically, by the two main traditions of the ancient Greek philosophy: the ones of Plato, and Aristotle together with their related declinations. But could a different interpretative key exist? I answer in the affirmative. The first one, more immediate, can be the one founded on the structure of the scientific text itself. But this is not easy and, above all, not univocally defined. The main risk is to project onto the ancient scientific thought the conception of the modern formalistic mathematics. A third way is to analyse such structure in the light of the entire historical, political, cultural and spiritual context in which the ancient scientists operated. The author employed such perspective in some previous researches[2] , of which the present paper is a natural development.

Before proceeding along this way, some notation is useful. I said that Euclid's texts do not contain explanations of the geometrical fundamental concepts. Such affirmation seems to be contradicted by some definitions that are present in the first book of the *Elements* and generally considered as explanations of the fundamental objects[3]. We observe, however, that the presumed definitions of fundamental geometric objects (*point, line, straight line, surface, plane surface, angle*, etc.), contained in the first book of *Elements*, do not really define anything. The related objects remain unspecified if they are not already known. It is clear and well known, in any case, that any linguistic system must start from some undefined terms. In ancient times too, I think, this was a real difficulty in approaching the study of geometry if Heron, introducing his *Definitions*, clarifies as follows the purpose of his work:

> ... to describe and draw for you, Dionysus, ... the technical terms that precede the *Elements* of Geometry. ... . In this way not only you will have a clear overview of his treatise but also of most of the other of those that devote themselves to the geometry [4].

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[2]Gentile-Migliorato [15],[16],[32]; Migliorato [29],[30],[31].

[3]The text that today is unanimously assumed as referring is the one edited by Heiberg [22]. The main English translation is in Heath [20]. The Italian edition by Acerbi [1] is particularly interesting for its critical apparatus.

[4]Translated from *Alexandrini Geometricorum et Stereometricorum Reliquiae*, edited by Fredericus O. Hultsh, Weidmannos, 1864. The original Grek text "*Και τά μεν προ της*

But in our case we have some definitions that are more indeterminate and obscure than the concepts which should be defined. This surely holds for the definition of straight line, but also for terms such as *point, plane surface, angle.* At that time this issue was discussed. In particular, the question was if Euclid considered his firsts definition as intuitionistic explanations of defined fundamental objects, coherently with Aristotelian thought. But, in this perspective, two orders of criticisms were advanced to the Euclidean system. The first one, which mentioned since the first century b.C., was that concerning the theory of the parallel straight lines and the fifth postulate. The second one properly regarded the nature of the fundamental geometric objects. The explanations about fundamental objects, indeed, are clearly inadequate to give an intuitionistic notion of them; in some cases they are even obscure and hard to decipher (this is the case of the straight line). We recall here, as an example, what Gino Loria said at the end of the nineteenth century:

> Euclide dà principio al proprio insegnamento geometrico esponendo una serie di proposizioni, alcune delle quali servono a dichiarare il significato dei termini tecnici, altre autorizzano a compiere certe operazioni geometriche, [...]. Al primo gruppo si possono muovere alcune critiche che, essendo di qualche gravità, non possiamo passare sotto silenzio: I. Parecchie definizioni mancano di chiarezza e precisione; tali sono quelle di punto linea in generale, linea retta e figura. [...] Tutto ci induce a supporre che il testo da noi posseduto delle definizioni abbia più del resto subito l'influenza modificatrice dei ricopiatori ... [5]

If the definitions (or explanations) under discussion were authentic then the criticisms moved would have been well-grounded, and we should ask ourselves the reason for which more than three centuries had passed before someone tried to bridge the lack. L. Russo argues, with a wealth of arguments [44], that the first seven definitions contained in the first book of Elemets are not originally written by Euclid, but have been added by later compilers. So the corresponding terms (*point, line, straight line, surface, plane surface*) would have originally been used without a definition or explanation, coherently with the fact that *Postulates* (*Αιτήματα*)

γεωμητρικής στοιχειάσεως τεχνολογοῦμενα", for the truth, can be itself interpreted in some different way. But the interpretation given, for example, by G. Giardina [17], that translate in Italian language "the technical terms that are *in beginning* of *Elements* of Geometry" (i termini tecnici che stanno all'inizio degli Elementi di Geometria), appear to me hardly acceptable. The shift in meaning of the Greek words *προ της* (first than) to denote the initial part of the Euclidean Work, is clearly finalized to justify the presence in the first book of *Elements* of some of such definitions and prove, in this way, their authenticity (see forward). But this pose serious problems. One, of these, is that only a little number of the Heron's definitions are really in the first book of Element, even if we admit their authenticity.

[5] "Euclid starts their geometric teaching exposing a series of propositions, some of which are used to declare the meaning of technical terms, others to do certain geometric operations, [...]. To the first group we can move several criticisms which, being of some gravity, we can not pass over in silence: I. Several definitions lack clarity and precision; these are those of point, line in general, straight line and figure. [...]. All this suggests that, more than other parts, the definitions have been subjected to the influence of the copyists" (Loria, [28] , pp 19-21).

and *Common Notions* ($K o\iota\nu\alpha\iota$ $\grave{\epsilon}\nu\nu o\iota\alpha\iota$), explicitly expressed at the beginning, are sufficient to prove all subsequent theorems.

I em inclined to accept this suggestion and I think that this may also apply to the definitions 8 and 9 (*angle* and *rectilinear angle*). The definition 8 introduces the notion of angle between any two lines; the definition 9 defines as rectilinear angle that formed by two straight lines. But in the rest of *Elements* the term *angle* is used always in the particular sense of *rectilinear*, with one exception: one single theorem in the third book. Such theorem is not essential and irrelevant for the general development. So, it is more reasonable to think that such theorem had been added in later period and Euclid had used the *angle* word as an undefined term.

2. **Explanations or undefined terms?**

If Heron felt the need to write a book of geometrical definitions, this is the sign that in the already existing texts, and particularly in Euclid's *Elements*, the correspondent terms were not conveniently explained. But if we suppose it, a question arises: why such problem was not posed in the past? How the contemporaries of Euclid and Archimedes understood the geometrical texts? The answer which I propose is the following[6]. The scientific activity can be conducted successfully only if one had an adequate research training in direct contact with a teacher, as a path of initiation to the forms of scientific thought. The entire educational system, from children's schools until the PHD, contribute today to achieving this target.

But this is true in all times. The fundamental transmission's schema of the basilar forms of thought can be found, for example, in *Menon* of Plato. Alexandria was certainly the main centre of Hellenistic culture, and it is a common notion that schools of all disciplines were active in the Museum. Recently F. Acerbi [1] puts into question if a mathematical school was supported as royal institution in the Museum of Alexandria. Certainly such question is histobrically important, but it is not decisive for us, because in any case many students spent some time in Alexandria in direct contact with the main scholars and scientists, independently from the existence of an institutional school, economically supported by the king. There is some reliable evidences of the presence of Archimedes and Apollonius in Alexandria and, for the second, in Pergamon too. The direct contact, in which the oral communication have an important and irreplaceable function, allows both the constitution of a scientific paradigm and his transmission in the time through generations. It is thus easy to understand that, if the direct oral transmission is

[6]Already in 1996 L. Russo [42] developed a wide and reasoned hypothesis according to which a true scientific revolution would have occurred at the beginning of the third century b.C., only to be forgotten afterwards at end of the Hellenistic kingdoms. I maintain that many of his arguments are well founded, but his explanation seems insufficient to us (see Migliorato [30], Chap. II and III). In particular Russo attributes the loss of the scientific Hellenistic thought to the estrangement and disregard towards it by Roman culture. Such explanation, I think, can only be partial, because in Roman Imperial period the scientific studies was actively cultivated in the centres of Greek language, and also supported by the Empire, but without important original developments.

suspended for some generations, then the original interpretation of the written texts can be lost and forgotten. The more a linguistic and conceptual system is abstract the more such loss is serious. This is certainly a very high risk for the conceptual foundations of mathematics. But from the age of Euclid and Archimedes and that of Heron, there was a considerable period of wars, looting, deportations and fallen empires. In 212 b.C., Syracuse was sacked and Archimedes killed. Around the middle of second century, the bloody feud in the family of the Ptolemies, to which the expansionist ambitions of Rome were no strangers, caused the flight of all the scholars from Alexandria. In 87 b.C. Athens was sacked by the troops of the Roman consul Cornelius Silla and many Athenian intellectuals were brought to Rome as slaves.

These are just a few examples, indicating an interruption of the processes of cultural transmission. The written scientific texts obviously remained. They have been taken up and studied by subsequent generations that tried to continue on the road already mapped out. But the meaning of its terms and its conceptual apparatus had to be rebuilt and could no longer coincide with the original one. This clearly explains the reason of the book of Heron on the geometrical definitions, and also the flourishing of a rich literature of comments on mathematical works, hitherto non-existent. It can also explain why, despite the considerable intellectual and economic efforts, no significant scientific advances were made at that time. This is not because brilliance and intellectual commitment lacked, but because, in the field of scientific research, any creative dimension and willingness to invent new conceptual categories had disappeared.

As we will see in the following, science proceeds successfully when its conceptual objects are produced as inventions of the mind, and inversely becomes sterile and unproductive when it claims to discover absolute truths about metaphysically given objects. The prevalent philosophical systems of such period, indeed, provided the main basis to the reinterpretation of scientific texts, prefixing the questions of metaphysics to the needs that arise from within the scientific research itself. Initially, the rediscovery of some of the themes of Aristotelian thought seemed to be prevalent. Later the Neo-Platonic thought played an increasing role.

A clear example is given by the *Commentary* of Proclus on the first book of Euclid's *Elements*. During the previous centuries, main philosophers too, such as Plato and Aristotle, faced the main scientific themes, but they did not add any significant contribution in terms of results and posed themselves in position of controversy and antagonism with "those who have acquaintance with geometry", as Plato says in the following passage:

> This at least [...] will not be disputed by those who have even a slight acquaintance with geometry, that this science is in direct contradiction with the language employed in it by its adepts. [...] Their language is most ludicrous [...] for they speak as if they were doing something and as if all their words were directed towards action. For all their talk is of squaring and applying and adding

> and the like, whereas in fact the real object of the entire study is pure knowledge[7].

As most of the scientific texts of antiquity are lost, the whole critical tradition on the ancient science was formed on the comments of the Roman imperial period. From this the myth was born of an ancient Greek science exclusively directed to the search for truth and ultimate essence of things. And it is singular that the history of mathematics and sciences continued to assume, until very recently, a substantial continuity. On the contrary, all other fields of history (from politics to literature, arts and philosophy) had highlighted fundamental discontinuities. In the following I will attempt to show some of the fundamental characters of ancient scientific thought, as they appear when freed from historically accumulated prejudices.

3. **The Euclidean Paradigm**

We conjectured already that the fundamental geometric terms were not explicitly defined or explained by Euclid. The understanding of the text, thus, could also be possible through an adequate educational training. During his training, the future scientist can acquire a shared heritage of linguistic and conceptual systems, procedures and notions accepted by the community of scientists. That is what Tomas Kuhn called a *Scientific Paradigm*. But where Euclid's Paradigm founded the meanings of its terms? How could one decide on the validity of its principles and its methods? In the Euclidean text there is not any explicit affirmation of existence and truth. He does not affirm that a straight line exists but only that from a point to another point a straight line can be traced. Better than an existence's affirmation, it seems to be the ideal translation of a material operation: exactly the inverse of what Plato required. The same can be said of the circle and the other geometrical entities. The entire geometrical system can be seen as an idealization of the real procedure to solve problems by rule and compass, and its fundamental terms appear as metaphoric names of undefined and unknowable objects, on which, however, it is possible to operate if some properties are postulated.

But we must not make the mistake to think Euclid's geometry as the modern axiomatic system[8]. In the latter case, indeed, the meaning of the terms is totally dissolved in a system of symbols and rules of use. In the Euclidean paradigm, as it appears in the *Elements*, instead, words seem to retain, through the process of metaphor, a strong semantic link with the objects from which they originated. A link that does not seem to be only of etymological or psychological nature, but that plays a role in the real way in which science grows and progresses. It has already been observed that in some step of the *Elements*, especially in the theory of magnitudes, the meaning will make up for any insufficiency of the postulates. Furthermore, in a modern axiomatic theory new terms and new axioms can be introduced arbitrarily and without any other restriction than that of logical consistency. In the Euclidean paradigm any new object seem to be the result of a

[7]Plato, *Republic*, 7, 527a-b. Transl. by J. Adam.

[8]See on this, Seidenberg [45]

new process of abstraction from already known objects by a metaphorical shift of meaning.

A significant example of such process will be presented below with the Archimedean concept of centre of gravity. In any case, in the Euclidean texts and in the mathematical works of the same period that we know, the truth of a scientific affirmation is never invoked. In the *Elements* we find two categories of fundamental propositions. One is that of the common notions (*Κοιναι ἐννοιαι*), that is propositions that, predictably, nobody wants to refuse. The other is that of the postulates (*ηιτήματα* = requests), that is propositions introduced by the expression ask for... (*ηιτήσθω*);.

This recall the typical form of the ancient Greek dialectic reasoning, in which the conductor of a dispute asked the listener to accept the premise. If they do, the interlocutors will be forced to accept the logical consequences too. The same can be said for the *Optics* in witch the word *υποκείσθω* (*let us suppose*) is used, to introduce the postulates. Furthermore, in this work there is not a distinction between introduction of terms and enunciation of postulates: instead, the technical terms are directly employed without explanation and the word *όροι* (*terms*) is used as title of the list of postulates. This is the sign that the postulates themselves are the only explanations or definitions of the technical terms. But what is *Optics*? I did not linger to discuss on interpretations that have been given in the past but now considered unreliable, such as that attributing to Euclid the acceptance of the Platonic theory of light. At the same time, the hypothesis of Panofsky, according to which the work was fundamentally a treatise on perspective[9], appears as insufficient to us. It seems, indeed, that the Euclidean Optics is an explicative theory of the vision's phenomena. The assumption of the visual cone as a discrete and finite set of straight lines, in fact, permits to explain not only the perspective vision but also the reduction of resolution power when the distance is increased. It is possible that the main purpose to which he aimed was the observation of the stars and its motions. And effectively the Optics is cited in the work that the our scientist dedicated to such topic, that is in the *Phenomena* .

> The fixed stars are seen always rising from the same place and setting in the same place, those rising simultaneously always rising simultaneously, and those setting simultaneously always setting simultaneously. Moreover, they are seen as always having the same distances from one another as they move from rising to setting. Since this happens only with [things] that move in circular paths, when the eye [of the observer] is equidistant in every direction from the circumference, as is shown in the Optics, one must postulate that the stars are carried in circles and are set into a body, and that the eye is equidistant from the [circles] circumferences[10].

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[9] Panofsky[40]

[10] Bergren, [3] , pp. 43-44. The translation is coherent with those ones in Latin language by Heiberg and Italian language by Acerbi [1].

There are not, here, affirmations on the true nature of the cosmos and the real movements of the stars, but only the hypothetical suppositions that appear more convenient to explain the phenomena as they appear. After this we can summarize the fundamental characters of the Euclidean paradigm as it appears from what was examined until now.

(1) Some non defined terms are used as technical terms. They are derived from previous knowledge, or practice of problem solving, or phenomenological data. Such technical terms are not used with their original meanings, but as metaphor of new ideal objects that are more properly defined in a technical apparatus by hypotheses or postulates.
(2) There is not any affirmation on the existence or truth of something. The validation of hypothesis can not be *a priori* but is necessaryly evaluated *a posteriori* because perfectly functional to explain the observed phenomena, or to solve problems, or to give a more complete and rational foundation to pre-existing traditions.
(3) The shared consent among experts is generally required as validation's condition of an hypothesis. Obviously the logical coherence of the reasoning is a necessary premise that can not ever be called into question.

This representation of the Euclidean Paradigm is obviously the result of a reconstruction and a reflection on the ancient texts which can be done today, in light also of the outcomes of modern scientific thought, nor it could be otherwise. The writer does not believe in absolute objectivity in history, therefore does not purport to assert definitive truth. But in proposing our own interpretation, we claim it as possessing a far greater level of consistency than others, taking into account the knowledge and the documents available today. The argumentations, that will be brought in the following to sustain this thesis, are the results of some of the researches I and other authors conducted on the topic in the last fifteen years.

4. **The Enigma of the Parallel Straight Lines**

The first sign of a misunderstanding about the Euclidean Paradigm of which we have notion, through the testimony of Proclus [41], is the attempt made by Posidonius, and then by Geminus, to prove Euclid's fifth postulate, as we learn from the *Commentary on the first book of Euclid's elements*. From this moment a long and enigmatic history began and continued until the starting of the nineteenth century, when the possibility of non-Euclidean geometries was recognized.

The reason for which they considered the Euclidean proposition inadequate to be used as a postulate was that it was not recognized as evident. But evidence is not among the conditions which can be found in the Euclidean Paradigm. The concept of evidence, in fact, is strictly tied with that of truth, and presupposes the search of an absolute and metaphysical knowledge. If we suppose, therefore, as for a log time was believed, that the search of metaphysical knowledge was an essential condition of the scientific discourse, then our reconstruction of the Euclidean Paradigm fails in whole or in large part. Why Posidonius, and others after him, maintained that the proposition of the fifth postulate must be proved? We can find an answer in the following passage of Aristotle:

> Therefore, if knowledge is that as we have established, than it will be necessary that the demonstrative science be built on the basis of true premises, first, immediate, best-known than conclusions, prior to it, and that causes it. [...] A syllogism could exist without these assumptions, but a demonstration could not exist, because then it would not produce science[11].

Here Aristotle seems to put a metaphysical causality's principle as basis of scientific deductive process, that is a natural hierarchy of relations, not conventional but intrinsic to the nature of the objects. The knowledge is not given until the absolutely primary and irreducible causes of objects are not recognised as immediate and irrefutable. On this way not a new scientific category can be introduced as free creation of the thought. This can explain why in the succeeding centuries there were a lot of commentaries in the sciences, but few real progresses and without new conceptual categories. The first century b.C., moreover, was that in which the esoteric works of Aristotle were discovered, ordered and, for the first time, published by Andronicus of Rhodes. The big metaphysical themes of knowledge were resumed, after the long and complex period of the Hellenistic thought. But in the centuries of Euclid, Aristarchus, Archimedes, Eratostenes, Herophilus, Apollonius, Ipparchus, there are not signs of metaphysical inclusions in the scientific discourses and, inversely, the extraordinary innovative fertility of this period constitutes a sign of their absence.

After the attempt of Posidonius, the problem of parallels straight lines was discussed for centuries in terms that are well known and was eliminated only in the nineteenth century when the asking of metaphysical evidence was substantially removed. And before Posidonius? Nobody appears to have posed problems on this matter, but in modern times the hypothesis was advanced of a presumed incertitude of Euclid himself. David Gregory, in his edition of *Elements* (1703), followed by many others, seems to have been the first to put such hypothesis. – Why – they say – Euclid did not use the fifth Postulate until the XXIX proposition? – This question seems to have a sense because some of previous propositions, that Euclid proves in a complex way, can be proved, using the fifth postulate, in a simpler way, as we do today in our schools. So they conclude that Euclid, doubting perhaps of his fifth postulate, would have used it only when absolutely needed.

I have widely discussed such argument in previous works, highlighting its total inconsistency. Before Euclid, indeed, as we can infer from several passages of Aristotle, the question of the uniqueness of parallel lines was not distinguished by that of their existence. Many times Aristotle poses such problem in terms of existence of parallel straight lines, or equivalent forms. Euclid solves the first twenty-eighth propositions using only the first four postulates, thus clearly distinguishing the two problems. The first one, regarding the existence, is solved inside the already existing apparatus, contrarily to the argumentations reported by Aristotle. For the

[11]Aristotle, *Anal. Post.*, 71B, 18-25.

second one, independent from the first and regarding the uniqueness, he gives a very genial solution by his fifth postulate. This appears to us a very strong reason for the choice of Euclid: any other way would have obscured the originality of his solution.

5. **Archimedes and the Euclidean Paradigm**

The image of Archimedes as an isolated genius is still widespread into the opinion of many scholars. They represent him as a person out of his time that aims his works only to people of future ages [12]. But sciences can be expressed only in an existent language and inside a conceptual apparatus that is already shared. To build a science for future people can not be true or false but simply meaningless. A scientist can indeed be deeper and sharper than the others, can innovate and anticipate new scenarios, and certainly Archimedes was the greatest scientist of his time. But he can not scientifically operate if not starting from an already existing and shared paradigm. And in his case, this must be what I called Euclidean Paradigm. It is not at all surprising that the myth of an alien mind inspired by gods would arise when, lost the key to scientific discourse, his results and his surprising realizations had to appear miraculous. But now, the coherence with a paradigm fundamentally assimilable to that of Euclid can be easily seen, if we read his works without any prejudice.

Firstly, the introductory letters appearing in some works can provide significant clarifications. Unlike Euclid, who worked directly in the Museum of Alexandria, Archimedes, like other contemporary scientists, could communicate with the rest of the Hellenistic world by sending his manuscripts in that city accompanied by a brief introductory presentation. It can be assumed, therefore, that Alexandria, as well as being the main training centre, irradiated knowledge through its library. It was a form, we could say, of publication. More than once, in these introductory presentations, Archimedes considers appropriate to clarify the reasons for his choices, but in no case he refers to truths inherent in the nature of things; in no case principles of causality are invoked or mentions can be found of supposed first causes. The reasons that justify his choice, on the contrary, are always referred to the use already made by other scholars, to the same confidence that they granted to the hypothesis, to the results obtained by their use, or to the possibility of obtaining new shared and shareable results.

This is the case, for example, of the postulate today known as *Eudoxus-Archimedes axiom*, that, that is: "given two greater (lines, surfaces or solid), a multiple exists

[12]The story handed down by Vitruvius, in which Archimedes comes out naked from the bathroom on the street crying "Eureka, Eureka" is known by all. The image that gives Raviel Netz is only apparently more credible when he says: "In most of Archimedes' letters there is a faint note of exasperation: there was no one to write to, no reader good enough. (There would be in time: Archimedes would eventually be read by Omar Khayyam, by Leonardo da Vinci, by Galileo, by Newton: these would be Archimedes' real readers and the ones through whom he made his real impact. He must have known that he was writing for posterity)." (Netz-Nöel, [36]).

of their difference that is greater of the bigger one". Archimedes justifies such assumption in the introductory letter of *Quadraturae Parabolae* and, implicitly, in *De Spaera et Cylindro*. For the first text, I should cite here at least the following passage of Archimedes himself that seem very eloquent to us:

> The earlier geometers have also used this lemma; for it is by the use of this same lemma that they have shown that circles are to one another in the duplicate ratio of their diameters, and that spheres are to one another in the triplicate ratio of their diameters, and further that every pyramid is one third part of the prism which has the same has similar to that aforesaid. And, in the result, each of the aforesaid theorems has been accepted no less than those proved without the lemma. As therefore my work now published has satisfied the same test as the propositions referred to, I have written out the proof and send it to you, first as investigated by means of mechanics, and afterwards too as demonstrated by geometry. Prefixed are, also, the elementary propositions in conics which are of service in the proof[13].

The case is far more subtle in the introductory letter of *De Spaera et Cylindro*. We could ask: such postulate is useful: But is it necessary?. Archimedes says:

> For, though these properties also were naturally inherent in the figures all alone, yet they were in fact unknown to all the many able geometers who lived before Eudoxus [that at first used a similar assumption], and had not been observed by any one. Now, however, it will be open to those who possess the requisite ability to examine these discoveries of mine[14].

We note now that the expression translated by Eath as "has been accepted" in the first of the cited passages, is expressed in the original text by the word *πίστιν* than can be translated as *confidence, faith, security or deep conviction*. From this A. Frajese argues that Archimedes had to have a platonic conception of Mathematics, because he required to have faith in the postulates and not assume it arbitrarily as in the modern mathematics [13]. But we have already excluded for the ancient mathematics the modern formalistic conception. We said that in ancient mathematics the objects, unlike those of formal systems, presuppose a meaning, even if mentally built as metaphor. Any scientist can have confidence on some presumed property of such objects, but the confidence is a personal and individual attitude that can be tied to its philosophical conception. So Archimedes, as Euclid, may have had its philosophical convictions, but they do not needed to express it in their scientific work. A scientific paradigm presupposes, inversely, a collective attitude of acceptance, regardless of the individual philosophical positions, that can be different. This is the only form of objectivity that science can achieve.

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[13]Transl. by Thomas Heath, [18], p. 234.
[14]Transl. by Thomas Heath, [19], p. 2.

6. **Archimedes and the Centre of Gravity**

Where the ideal scientific concept comes from? This question seems to recall the title of a recent book of Lakoff and Núñez [26] that proposes an interesting analysis on the cognitive processes by which the main mathematical concepts are generated in our mind. But our purpose is different, even if the concept of metaphor can be useful for us. By the word *metaphor* we intend, coherently with Lakoff, a term that in the usual language denote a specific and known object, but in the given specific context is used to express an ideal and abstract object that preserves some characters of the original one. It is not possible, today, to carry out a detailed historical analysis of concepts such as *point, line, circle, surface*, but it is easy to imagine them as derived from the resolution of technical problems by drawing lines on paper. The Euclidean geometry is, I think, a result of operating with ruler and compass in the same way in which the fractal geometry is a result of operating with computers. But if the origin of geometric concepts as *point* and *line* are lost in remote times, for the Archimedean concept of *centre of gravity* we can find precise traces in his work de *Planorum Aequilibris* (on the equilibrium of planes). Traditionally this work posed a problem. Because the *centre of gravity* is used but not defined, several scholars have supposed the existence of a precedent work, now lost, in which such definition was given[15]. But, as we discuss in other publications[16], not only no reference is known to such work, but a realistic definition of the concept is not possible. Inversely, it remains perfectly operatively defined by the postulate that opens the text and, moreover, it assumes a typically geometric character. I summarise here briefly the reasons for this claim.

Before Archimedes, the more similar notion used in mechanics was that of *centre of suspension* of a balanced object. It can be easily defined, in realistic and observational terms, as the point by which a pole with weights at the ends must be suspended to remain horizontal. We can easily consider the centre of gravity as a generalization of such notion to the case in which the weights are not in two points, at the end of a pole, but uniformly distributed on a given geometrical figure. But in such case, the idea of suspension can be absurd when the centre is out of the figure or inside a material solid object. The new notion becomes possible, instead, when it is seen as an ideal abstraction from the previous one. In such meaning it becomes perfectly defined if the rules to determine it are given for each case. Archimedes obtains this by his postulates.

The ideal character of the new concept, introduced here by Archimedes, is moreover enforced because the entire work assumes as its object plane figures that do not have a weight. His geometrical character is given, in particular, by the postulate 3 that establishes: "When equal and similar plane figures coincide if applied one another, their centres of gravity coincide".

So, given the geometrical characters of a figure, his centre of gravity is determined and no physical condition can change it. Moreover, the term weight is always identified with a magnitude applied in a point. Such magnitude, when it is specified,

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[15] See for ex. Sato [44]

[16] Migliorato [31], Gentile-Migliorato [15] [16].

is the area or volume of a geometrical figure and the point of application is the centre of gravity. In other words, we can assume the term *equilibrium*, independently from its physical meaning, saying that a geometrical figure is in equilibrium with respect to a point (said suspension point), if and only if it is its centre of gravity. Only when we give to the term *equilibrium* its physical meaning, then the empirical observation of the world becomes necessary, otherwise these notions remain purely geometric even if they are suggested by the observation of the world and retain from this the original language.

In the subsequent postulates and consequent theorems, the premises are posed for the use of such notions both in geometrical and in mechanical problems. This is obtained using generically the term *magnitude* that can be indifferently interpreted as *area*, *volume* or *weight*.

7. Mathematics and Real World

The notion of centre of gravity has an important application to the real world in the work *On the Floating Bodies*. The second book of this work, in fact, studies the conditions of equilibrium of a floating body immersed in a liquid. It is significant, in our opinion, that the solid on which the work is almost entirely centred, is a segment of paraboloid, the section of which is similar to that of the hull of a ship. This suggests that precise balance problems of ships have inspired that work. A confirmation can be seem in some reports that Archimedes had a role in overseeing the construction of a ship of unusual dimensions.

If we accept this interpretation, then it means that, contrarily to the tradition starting with Plutarchus, Archimedes had a real interest in the technological applications and did not exclude it as a model and, in some cases, as a goal of his investigations. In any case the explanation of the real world, as it appears, was the object of the scientific paradigm adopted by scientists of this period. An observation to this effect can be made on the same work of Archimedes that we have just considered. In the first book, in fact, he assumes that weights are directed to the centre of the Earth, and starting from this he proves that the surface of the sea is spherical. In the second book, having to investigate a small region of space, he assumes that the weights are parallels. This, obviously, is correct as approximation in the considered situation, but only if we want to explain some phenomenon, not if we want to express a metaphysical truth.

In the same way, as we have previously seen, Euclid mentioned explanations of phenomena such as the motions of the stars or the perspective vision. Along this route Aristarchus must have proceeded when he proposed a cosmological model in which the Sun is at the centre of the system. Such model is assumed by Archimedes in the *The Sand Reckoner*. The object of the work is the effective possibility to build very large numbers. So he calculates the number of grains of sand necessary to fill a sphere having the size of the universe and, to do that, chooses the model of Aristarchus.

The motivation is very interesting, because he does not try to determine which is the true model, but chooses the one that requires the larger size. Thus, In this case too, the choice of an assumption is determined not by a presumption of absolute truth, but by its usefulness in solving a problem. We have, however, strong reasons to maintain that Archimedes had chosen Aristarchus's system also for properly cosmological problems. We know, in fact, that he had built a planetary able to accurately represent eclipses and oppositions of planets. Brought to Rome, it was described by Cicero in a passage of *Republic*, in which he refers that Sulplicius Gallus, expert in astronomy, while explaining to him the functioning of the device, affirmed substantially that no human mind could understand as one single motion could produce the different and complex motions of sun, moon and the various planets[17]. Afterwards, Cicero himself had written that Archimedes could not get that result if not guided by a divine mind[18]. Today we know that such result can only be obtained if a model is used that is centred on the sun[19].

This can mean two things: Archimedes had used the model proposed by Aristarchus but at the time of Cicero this was inconceivable. Naturally other hypothesis could be possible but, in any case, it seems clear that the choice of assumptions was made not on the basis of beliefs about reality but in order to obtain acceptable results and to explain phenomena.

8. **The infinity**

It is well known that the Euclidean geometric system carefully avoids every statement that presupposes the existence of infinite quantities. So the possibility to extend a straight line replaces the concept of the infinite line that we use in modern mathematics. In this way the more difficult problem was the one connected with the theory of parallel straight lines. Euclid solved it in a very genial way, by the fifth postulate, establishing the property of parallel straight lines without mentioning it. But the problem was destined to re-emerge, and many passages of ancient mathematical thinking were born out of the attempt to remove it.

I devoted two chapters of a book [30] to this problem in Archimedes but, because it is only published in Italian , I recall here some of its essential points. In *The Method*, various geometrical theorems are obtained at first by mechanical way and after proved by the geometrical method called *of exhaustion*, using the Eudoxus-Archimedes postulate. The reason why Archimedes proposes his mechanical method to find solutions for geometrical problems is easy to understand and is expressed by the author himself in his introductory letter to Eratosthenes, where he writes:

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[17]Cicero, *De Re Publica*, I, 21-22.

[18]"nam cum Archimedes lunae solis quinque errantium motus in sphaeram inligavit, effecit idem quod ille, qui in Timaeo mundum aedificavit, Platonis deus, ut tarditate et celeritate dissimillimos motus una regeret conversio. quod si in hoc mundo fieri sine deo non potest, ne in sphaera quidem eosdem motus Archimedes sine divino ingenio potuisset imitari" (Cicero, *Tusculanae Disputationes*, I, 63).

[19]As Neugebauer noted at first in 1975 [39].

> I thought fit to write out for you and explain in detail in the same book the peculiarity of a certain method, by which it will be possible for you to get a start to enable you to investigate some of the problems in mathematics by means of mechanics. This procedure is, I am persuaded, no less useful even for the proof of the theorems themselves; for certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because their investigation by the said method did not furnish an actual demonstration. But it is of course easier, when we have previously acquired, by the method, some knowledge of the questions, to supply the proof than it is to find it without any previous knowledge[20].

This becomes clearer if we consider that the procedure of exhaustion used by Archimedes to prove such theorems is applicable only when the result is already known, or at last suspected, but it is not useful to find it. It is fully justified, so, the search for an heuristic way in order to gain some information on what I try to prove. But why does he maintain that "the said [mechanical] method did not furnish an actual demonstration"? If we continue to assume Archimedes to be a Platonist, than the answer can appear, at first sight, very easy: mechanical notions belong to the material world so they are inadequate to prove eternal and transcendent geometrical truths.

But another question inevitably arises, in this case: Archimedes have used mechanical method to prove geometrical theorems in other cases. In *Quadratura Parabolae* too he proved the same theorem at first following a mechanical path and afterwards by exhaustion. But in that case, he did not question the effectiveness of the first demonstration. Indeed, after having used mechanical methods, he began the 17^{th} *Proposition* by the words "*Τούτου δειγμένον φανερον* ..." that Heiberg translates in Latin language as "Hoc demonstratum manifestum est,..." which we can read as "This being proved it is evident that ... ". It remains the fact that Archimedes proves the same theorem in another way, but we can explain this by the fact that the mechanical way requires the acceptance of some terms and assumptions that were not part, at the time, of a shared paradigm. The question also remains: why should Archimedes consider as proved the propositions contained in *Quadratura Parabolae* and not those that he communicated to Eratosthenes in *Methods*, if both are obtained by mechanical way? I em aware that any conclusion in this respect is a risk, because of the temporal distance, the lack of available material, the philological difficulty and the possible corruption of texts. This is the reason why I believe that the historical reliability is to be evaluated not so much on individual *Elements* but, more importantly, for the coherence of the global framework.

That said, it seems to us that, whatever the thought of Archimedes, a substantial difference between the two cases can be found, and this difference is in the way of considering the infinity. In *Quadratura Parabolae*, even if the demonstrative route is

[20] Ttransl. by Thomas Heath, [18], p. 13.

long and complex, however the use of notions as equilibrium and centre of gravity is determined exclusively by the postulate already set in *De Planorum Aequilibris* and substantially reducible to the usual geometrical field. In Method the geometrical substantial character is maintained, but such supplemental assumptions determine a new, and still unexplored, situation. Consider, for example, the lemma 3 that says
– *If the centres of gravity of magnitudes as large as wanted are on the same line, the centre of gravity of all will be on the same line.*

This lemma, derivable from precedent works such as *De Planorum Aequilibris* does not contain, by itself, anything dangerous, as long as the considered magnitudes are a finite number or indeterminate. It became an actual infinity when the lemma is applied to all the parallel sections of a plane or solid figure. Procedures of such type are normally used by Archimedes in the *Method* and introduce, for the first time in antiquity, the actual infinity in mathematical reasoning. This is, by itself, a good reason to consider it necessary to prove in another way the results thus obtained. We know that, in modern times, the infinitesimal methods of indivisibles adopted by Cavalieri and Leibniz is derived from this, but at that time it was not a shared and accepted paradigm. Could have it become one? We'll never know. What we know is that after some time the crisis of Hellenistic science began and any scientific paradigm was lost.

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